The DPG* Method

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(joint work with Leszek Demkowicz, Jay Gopalakrishnan)

We present a novel framework for the construction and analysis of finite element methods with trial and test spaces of unequal dimension. At the heart of this work is a new duality theory suitable for variational formulations with non-symmetric functional settings [5]. The primary application of this theory, in this talk, is the development and analysis of discontinuous Petrov–Galerkin (DPG) finite element methods; in particular, goal-oriented adaptive mesh refinement strategies therein.

We introduce the DPG* finite element method [3]: the dual to the DPG method. DPG, as a methodology, can be viewed as a practical means to solve overdetermined discretizations of boundary value problems. In a similar way, DPG* delivers a methodology for underdetermined discretizations. Supporting this new finite element method are new results on a priori error estimation and a posteriori error control. Notably, it is demonstrated that the convergence of a DPG* method is controlled, in part, by a Lagrange multiplier variable which plays the role of the solution variable in DPG methods. The presented theory is applied to two representative problems coming from linear and nonlinear partial differential equation models. To facilitate a thorough mathematical analysis, Poisson’s equation is considered. To demonstrate the utility of the approach in less tractable scenarios, the Oldroyd-B fluid model is also considered. Taken together, the combined analysis of these two models effectively demonstrates the utility of the newly developed paradigm. Taken together, the combined analysis of these two models effectively demonstrates the utility of the newly developed paradigm. This talk is largely based on Keith’s Ph.D. dissertation [2].

References